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Experiment with Water in a Tumbler - NAEST 2020

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Problem Statement:

In this experiment you will explore the relation between u and v for a thick convex lens. It will be water in a cylindrical vessel/tumbler or pot. Though the lens so formed is not spherical, but if the rays are collected in a *horizontal* plane, one can still use u- v relations and see the effect of the thickness of the lens.

(Experiment in Prelims of National Anveshika Experimental Skill Test 2020)

1. Discussion in the Prelims Committee (8 Members) chaired by Prof H C Verma

The Online Examination and Experimentation at home, leisurely in 60 hours, gave the experimenter (student) time to think, plan and do. The committee members identified the issue and were clear about what needs to be done. Of course, we had different views of the same problem and worked in our own style. We came up with a variety of results and many different strategies to achieve the goal.

During our discussions, we put down our differences and listened carefully to understand each other. We did a lot of brainstorming and gave lots of room for everyone's creativity. We separated the many options and evaluated the options with varying weightage. Weighed all the pluses and minuses and came out with an optimum evaluation scheme with lots of room for the student's creativity and ingenuity. We had to 'bundle' a lot of options to come to the satisfactory solution.

Documentations was done faithfully to jot down all the details and implications. And the best part, there was plenty of room to monitor, follow-through and change wherever required!

2.Question to be addressed?

Our inference at the end of our experimentation was divided, was the focal length (determined for our thick water lens) a ***constant*** or ***varying*** with the distance of the object?

Guess what, working out the theory gave us the answer.

3.Theory:

a. Relation between u, v, f

Sign Convention is the Cartesian Coordinate System. Origin is H for the thick lens. For refraction at the first and second surfaces, V_1 and V_2 are the origins, respectively.

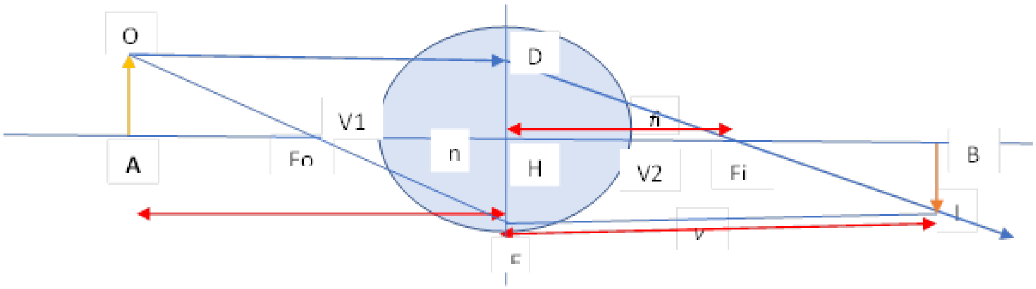

Fig 1. Ray diagram of a circular lens (blue) . OA and BI are the object and image respectively. Fo and Fi are the foci. u and v are the object and image distances. All measurements are from DE.

In Fig.1, object OA produces an image IB. The object and image distances for the thick lens is measured from the principal plane DE.DE cuts the principal axis AB at H. For a circular lens, the two principal axes D_1H_1 and D_2H_2 are coincident and are along diameter of the circle (DH) of radius R (due to symmetry). That is, H_1 and H_2 are at the centre of the circle H. V_1 and V_2 are the vertices of the thick lens. Radius of curvature of the left surface is $R_1 = R$ and the right surface $R_2 = -R$

OD = object distance = $-u$

EI = image distance = v

HF_i = second focal length of the lens = fi

Ray diagram. A ray (OD) parallel to the principal axis emerges through the second Focus F_i of the lens as DF_i. Another ray (OF_o) from O passing through the first focus F_o emerges as EI, parallel to the principal axis. The two emergent rays met at the point I to give the conjugate image position I of the object O.

Let the object height be y_o and the image height be y_i. Consider the similar triangles OF_oA and EF_oH

$$\frac{y_o}{y_i} = \frac{AFo}{HFo} = \frac{u - f_o}{f_o} \quad \ldots\ldots \qquad (1A)$$

In the similar triangles DHF_i and F_iIB

$$\frac{y_o}{y_i} = \frac{HF_i}{F_iB} = \frac{f_i}{v - f_i} \quad \ldots\ldots. \qquad (2A)$$

If both the image space and the object space have the same RI (air in our case), the focal lengths are equal.

Therefore, $f_o = f_i = f$

From Equations (1A) and (2A)

$$uv - vf - uf + f^2 = f^2$$

$$\textit{Dividing with } uvf \textit{ throughout}, \ldots\ldots. \qquad (3A)$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Considering the cartesian coordinate Sign convention, *u* is negative while *v* and *f* are positive.

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \ldots\ldots. \qquad (4A)$$

This is the lens equation of the thick lens where *f* is the ***effective focal length***, measured from the principal plane.

b. To find the relation between the focal length and the radii of curvatures of a system of curved surfaces separated by a distance *d* from the vertices V_1 and V_2 ---

Consider two curved surfaces $D_1V_1D_2$ and $D_1V_2D_2$ where D1D2 is the diameter of the circular lens system. The two curved surfaces separated by 2R and RI = *n*, is ***equivalent*** to the thick lens above,

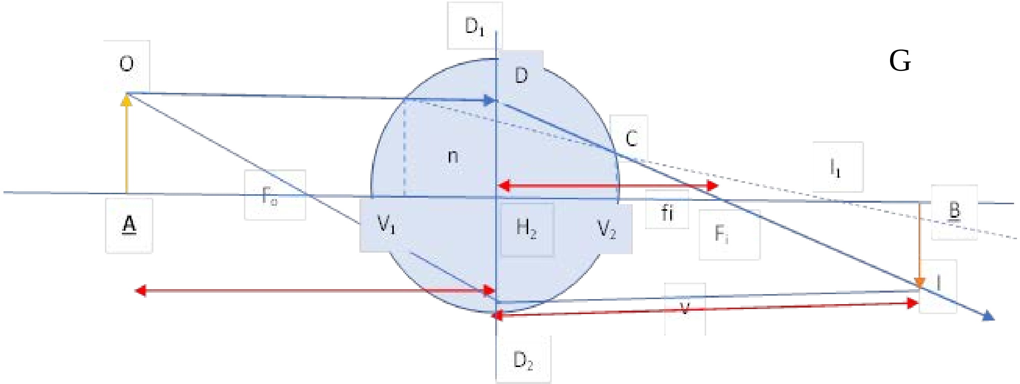

Fig.2. Relation of radii of curvatures and focal length fi of a thick lens.

provided a parallel beam of light from the left passes through the second focal length F_i of the above lens (or vice versa).

For refraction at the first surface, all measurements are done from V_1. Sign convention is the Cartesian Coordinate system.

OD and AB are rays parallel to the principal axis and can be considered to come from infinity and is incident on the first surface of radius R_1(= R). OG and AV_1 converge at I_1 (then I_1 is the focal point of the first refracting surface). Therefore,

$$-\frac{1}{\infty}+\frac{n}{V_1I_1}=\frac{n-1}{R_1}\text{.......(1)}$$

GC refracts at the second surface ($D_1V_2D_2$) and finally meets at Fi. The radius of curvature of the second surface is R_2 = -R. The distance between the two refracting surfaces is V_1V_2 = d = 2R.

Therefore, the virtual object at I_1 forms a real image at F_i and the relation due to refraction is

$$-\frac{n}{V_2I_1}+\frac{1}{V_2F_i}=\frac{1-n}{R_2}\text{.......(2)}$$

$$V_2I_1=V_1I_1-V_1V_2$$

$$V_1V_2=d$$

$$\frac{V_2I_1}{V_1I_1}=1-\frac{d}{V_1I_1}\text{.......(3)}$$

From similar triangles DH_2Fi and CV_2Fi

$$\frac{yo}{yi} = \frac{H_2Fi}{V_2Fi} = \frac{fi}{V_2Fi} ....(4)$$

From similar triangles GV_1I_1 and CV_2I_1

$$\frac{yo}{yi} = \frac{V_1I_1}{V_2I_1} ....(5)$$

From (4) and (5)

$$\frac{1}{fi} = \frac{V_2I_1}{V_1I_1} \cdot \frac{1}{V_2F_i} .....(6)$$

Using Equations (1), (2), (3), (6)

$$\frac{1}{fi} = \frac{V_2I_1}{V_1I_1} \cdot \frac{1}{V_2F_i}$$

$$\frac{1}{fi} = \frac{V_2I_1}{V_1I_1} \cdot \left(\frac{1-n}{R_2} + \frac{n}{V_2I_1} \right)$$

$$\frac{1}{fi} = \frac{V_2I_1}{V_1I_1} \frac{1-n}{R_2} + \frac{n}{V_1I_1}$$

$$\frac{1}{fi} = \left(1 - \frac{d}{V_1I_1} \right) \frac{1-n}{R_2} + \frac{n}{V_1I_1}$$

$$\frac{1}{fi} = \frac{1-n}{R_2} - \frac{d}{V_1I_1} \frac{1-n}{R_2} + \frac{n}{V_1I_1}$$

$$\frac{1}{fi} = \frac{1-n}{R_2} - \frac{1}{V_1I_1} \left(\frac{1-n}{R_2} d - n \right)$$

$$\frac{1}{fi} = \frac{1-n}{R_2} - \frac{n-1}{nR_1} \left(\frac{1-n}{R_2} d - n \right)$$

$$\frac{1}{fi} = \frac{1-n}{R_2} + \frac{(n-1)^2 d}{nR_1R_2} + \frac{n-1}{R_1}$$

$$\frac{1}{fi} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2} \right] .....(7)$$

The focal length is dependent on the refractive index of the media and the radius of the curved surfaces. Thus, for a given system of fixed *n* and R_1 and R_2, ***the focal length is a constant.***

Since OD the incident ray (parallel to the principal axis) meets the emergent ray DFi at D, DH_2 must be the ***principal plane*** of the thick lens so formed and Fi is the ***second focus*** of the thick lens.

The focal length of the thick lens is $HF_i = f_i = f$ as in Fig 1.

Therefore, substituting Equ. (7) in (4A) gives

$$-\frac{1}{u}+\frac{1}{v}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n-1)d}{nR_1R_2}\right] .....(8)$$

This is the ***lens equation of the thick lens***.

For a ***cylindrical lens***, R_1 = R, and R_2 = -R , hence

$$\frac{1}{f_i}=\frac{n-1}{n}\frac{2}{R}$$

Of course, the distance between the vertex and the principal planes are $V_1H_1 = h_1$ and is positive and $H_2V_2 = h_2$ is negative as shown in the figure and is given by (not shown explicitly here),

$$h_1=-\frac{f_i(n-1)d}{nR_2} \quad and \quad h_2=-\frac{f_i(n-1)d}{nR_1}$$

Eg. For the **cylindrical lens**

R_1 = R, R_2 = -R, d = 2R

$$h_1 = R$$
$$h_2 = -R$$

from V_1 and V_2, the poles of the curved surfaces. And both the ***principal planes*** are ***coincident*** and are along the diameter, perpendicular to the principle axis.

Hence our measurements of object distance and image distance from the centre, for the experiment, is perfect. What we are measuring as 1/|u| and 1/|v|, and the sum is giving us the ***1/f*** of the thick lens and should be a ***constant.***

4. Inference

A sample result is quoted here

- With **glycerine** (RI 1.473) filled in the tumbler,

a. Experimental value of $\frac{1}{|u|}+\frac{1}{|v|}=0.210cm^{-1}$

b. Lens makers formula for a thin lens is $\frac{1}{f}=(n-1)\frac{2}{R}=0.328cm^{-1}$

c. Lens makers formula for thin lens is $\frac{1}{f}=\frac{(n-1)}{n}\frac{2}{R}=0.219cm^{-1}$

d. Experimental focal length = (4.75 ± 0.07) cm

e. Focal length from lens makers formula for thin lens is (3.05 ± 0.05) cm

f. Focal length from lens makers formula for thick lens is (4.57 ± 0.07) cm

- With **water** (RI =1.33) filled in the tumbler

g. Experimental focal length = 6.45 cm ± 2%

h. Focal length from lens makers formula for thin lens is 4.63 cm ± .6%

i. Focal length from lens makers formula for thick lens is 6.13 cm ± 1.6%

- RI of glycerine measured after replacing water with glycerine in the same tumbler

j. RI of glycerine is 1.50 ± 0.05

k. Standard value of glycerine is 1.473

5. Conclusion

The focal length of the water-filled-tumbler-lens matched the les makers formula of a thick lens and was a constant.

The revelation was quiet thrilling. The joy of doing experiments with daily use gadgets was satisfying.

References:

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